General Covariance in Gravity at a Lifshitz Point

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Contents

Gravity may be the one force of nature we are intuitively most familiar with, but its theoretical understanding – despite the beauty of general relativity and string theory – is still shrouded in surprisingly many layers of mystery. Perhaps we already have all the pieces of the puzzle, and just need to find the correct way of putting them together; or perhaps new ideas are needed. In this context, the idea of gravity with Lifshitz-type anisotropic scaling [1–3] has attracted a lot of attention recently.

In Part 1 of this paper, we briefly review some of the main features of quantum gravity with anisotropic scaling, in its original formulation initiated in [1–3], and comment on its possible relation to the causal dynamical triangulations (CDT) approach to lattice quantum gravity. Part 2 explains the construction of gravity with anisotropic scaling with an extended gauge symmetry – essentially a nonrelativistic version of general covariance – presented in [4]. This extra symmetry eliminates the scalar graviton polarization, and thus brings the theory closer to general relativity at long distances.

1 Gravity with Anisotropic Scaling

The central idea of [1, 2] is a minimalistic one: to formulate quantum gravity as a quantum field theory, with the spacetime metric as the elementary field, in the standard path-integral language. Quantum field theory (QFT) has emerged from the 20th century as the universal language for understanding systems with many degrees of freedom, ranging from high-energy particle physics to condensed matter, statistical physics and more. Before giving up on QFT for quantum gravity, it makes sense to apply its full machinery to this problem, without prior restrictions such as microscopic relativistic invariance. The novelty of [1, 2] is that gravity is combined with the idea of anisotropic scaling of spacetime, more familiar from condensed matter, and characterized by

$$\mathbf{x} \to b\mathbf{x}, \qquad t \to b^z t. \tag{1.1}$$

Here z is an important observable, the "dynamical critical exponent," associated with a given fixed point of the renormalization group (RG). Systems with many different values of z are known, for example in dynamical critical phenomena or quantum criticality. It is natural to ask whether one can construct theories with anisotropic scaling and with propagating gravitons. Why? A consistent theory of gravity with anisotropic scaling can be potentially useful for a number of possible applications:

- (i) Phenomenology of gravity in our Universe of $3 + 1$ macroscopic dimensions.
- (ii) New gravity duals for field theories in the context of the AdS/CFT correspondence; in particular, duals for a broader class of nonrelativistic QFTs.
- (iii) Gravity on worldsheets of strings and worldvolumes of branes.
- (iv) Mathematical applications to the theory of the Ricci flow on Riemannian manifolds [1].
- (v) IR fixed points in condensed matter systems, with emergent gravitons (new phases of algebraic bose liquids) [5].
- (vi) Relativistic gravity and string theory in asymptotically anisotropic spacetimes [6];

and possibly others. Note that only application (i) is subjected to the standard observational tests of gravity, while the others are only constrained by their mathematical consistency. And of course, applications (i–vi) aside, this system can serve as a useful theoretical playground for exploring field-theory and path-integral methods for quantum gravity.

This approach shares some philosophical background with the idea of asymptotic safety, initiated in [7] and experiencing a resurgence of recent interest. Both approaches are equally minimalistic, suggesting that gravity can find its UV completion as a quantum field theory of

the fluctuating spacetime metric, without additional degrees of freedom or a radical departure from standard QFT. While both approaches look for a UV fixed point, they differ in the nature of the proposed fixed point: In asymptotic safety, one benefits from maintaining manifest relativistic invariance, and pays the price of having to look for a nontrivial, strongly coupled fixed point. In gravity with anisotropic scaling, one gives up Lorentz invariance as a fundamental symmetry at short distances, and looks for much simpler, perhaps Gaussian or at least weakly coupled fixed points in the UV. The price to pay, if one is interested in application (i), is the need to explain how the experimentally extremely well-tested Lorentz symmetry emerges at long distances.

Such Gaussian fixed points of gravity with $z > 1$ can also serve as IR fixed point in condensed matter systems, as shown for $z = 2$ and $z = 3$ in [5]. This may be important because it leads to new phases of algebraic bose liquids, and gives a new mechanism for making gapless excitations technically natural in condensed matter. Implications for quantum gravity are less dramatic: The gapless excitations at the IR fixed points of [5] are linearized gravitons, only allowed to interact in a way which respects linearized diffeomorphism invariance. Hence, this lattice model is not a theory of emergent gravity with nonlinear diffeomorphism symmetries.

1.1 The minimal theory

In our construction of Lifshitz-type gravity, we assume that the spacetime manifold M carries the additional structure of a codimension-one foliation $\mathcal{F}$, by D-dimensional leaves Σ of constant time. We will use coordinate systems $(t, \mathbf{x} \equiv x^i)$, $i = 1, \ldots D$, adapted to $\mathcal{F}$.

Perhaps the simplest relevant example of systems with Lifshitz-type anisotropic scaling is the Lifshitz scalar theory with $z = 2$,

$$S = \frac{1}{2} \int dt \, d^D\mathbf{x} \left\{ \dot{\phi}^2 - (\Delta\phi)^2 \right\}, \tag{1.2}$$

with Δ the spatial Laplacian. Compared to the relativistic scalar in the same spacetime dimension, the Lifshitz scalar has an improved UV behavior. The scaling dimension of ϕ changes to $[\phi] = (D - 2)/2$, and conseqently the (lower) critical dimension also shifts, from the relativistic $1 + 1$, to $2 + 1$ when $z = 2$.

The most "primitive" theory of gravity similar to (1.2) would describe the dynamics of the spatial metric $g_{ij}(\mathbf{x}, t)$, invariant under time-independent spatial diffeomorphisms. Because of the lack of (time-dependent) gauge invariance, this model would propagate not only the tensor polarizations of the graviton, but also the vector and the scalar. This "primitive" theory becomes more interesting when we make it gauge invariant under foliation-preserving diffeomorphisms $\mathrm{Diff}(M, \mathcal{F})$, generated by

$$\delta t = f(t), \qquad \delta x^i = \xi^i(t, \mathbf{x}). \tag{1.3}$$

The minimal multiplet of fields now contains, besides g_{ij}, also the lapse function N and the shift vector N_i. Since the lapse and shift play the role of gauge fields of $\mathrm{Diff}(M, \mathcal{F})$, we can assume that they inherit the same dependence on spacetime as the corresponding generators (1.3): While $N_i(t, \mathbf{x})$ is a spacetime field, $N(t)$ is only a function of time, constant along Σ. Making this assumption about the lapse function gives to the minimal theory of gravity with

anisotropic scaling, sometimes referred to as the "projectable" theory [2]. (For its brief review and some phenomenological applications, see [8].)

The dynamics of the projectable theory is described by the most general action which respects the $\text{Diff}(M, \mathcal{F})$ symmetry. At the lowest orders in time derivatives, the action is given by

$$S = \frac{2}{\kappa^2} \int dt \, d^D\mathbf{x} \, \sqrt{g} \, N \left(K_{ij} K^{ij} - \lambda K^2 - \mathcal{V} \right), \tag{1.4}$$

where

$$K_{ij} \equiv \frac{1}{2N} \left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right) \tag{1.5}$$

is the extrinsic curvature of Σ, $K = g^{ij} K_{ij}$, λ is a dimensionless coupling, and the potential term $\mathcal{V}$ is an arbitrary $\text{Diff}(\Sigma)$-invariant local scalar functional built out of g_{ij}, its Riemann tensor and the spatial covariant derivatives, but no time derivatives.

Which terms in $\mathcal{V}$ are relevant will depend on our choice of z at short distances. Terms with $2z$ spatial derivatives have the same classical scaling dimension as the kinetic term, and their quadratic part defines the Gaussian fixed point. Terms with fewer derivatives represent relevant deformations of the theory. They induce a classical RG flow, which can lead to an IR fixed point, with the isotropic $z = 1$ scaling in the deep infrared regime. As usual in effective field theory, terms of higher order in derivatives, or involving additional time derivatives, are of higher dimension and therefore superficially irrelevant around the UV fixed point.

Compared to general relativity, the minimal model is different in three interconnected ways: It has one fewer gauge symmetry per spacetime point, its field multiplet has one fewer field component per spacetime point (since N is independent of x^i), and it propagates an additional scalar graviton polarization in addition to the standard tensor polarizations, at least around flat spacetime. While the number of gauge symmetries and field components may not be observable, the number of propagating graviton polarizations is.

1.2 The nonprojectable case

Another possibility is to insist on matching the field content of general relativity, and promote the lapse N to a spacetime field. This is the "nonprojectable" theory [1, 2]. If we postulate the same $\text{Diff}(M, \mathcal{F})$ gauge symmetry as in the projectable case, the generic action will contain new terms, constructed from the new ingredient $a_i \equiv \partial_i N / N$. The general theory with such new terms is sometimes referred to in the literature as the "healthy extension" of the projectable theory. This is a misnomer – indeed, the basic rules of effective field theory clearly instruct us to include all terms compatible with the postulated symmetries, since such terms would otherwise be generated by quantum corrections. Hence, including all terms compatible with the gauge symmetry should not be called a "healthy extension" of the nonprojectable theory; it is just the correct implementation of the assumptions of the nonprojectable theory. (For a recent review of the nonprojectable theory, see [9].)

In contrast, leaving the a_i-dependent terms artificially out deserves to be called an "unhealthy reduction" of the projectable theory. Such an unhealty reduction could only be justified if it is protected by additional symmetries. However, a closer analysis of the unhealthy reduction indeed reveals difficulties with the closure of the constraint algebra and no

new gauge symmetry [10, 11], possibly with the interesting exception of the deep infrared limit [12].

The nonprojectable model may be described in terms of the same field content as general relativity, but the scalar graviton polarization is still present in its physical spectrum. In Part 2, we discuss a mechanism proposed in [4], which eliminates the scalar graviton, by enlarging the gauge symmetry to "nonrelativistic general covariance."

1.3 Entropic origin of gravity?

There is another concept originally introduced in [1, 2] which has caused some level of confusion in the literature: the "detailed balance" condition. This concept has its roots in nonequlibrium statistical mechanics and dynamical critical phenomena. Oversimplifying slightly, the theory is said to be in detailed balance if the potential in (1.4) is of a special form, effectively a square of the equations of motion associated with a (Euclidean-signature) theory in D dimensions with some action W. For example, the Lifshitz scalar (1.2) is in detailed balance, with $W = \frac{1}{2} \int d^D \mathbf{x} \, \partial_i \phi \partial_i \phi$.

In [1, 2], this condition was suggested simply as a technical trick, which can possibly reduce the number of independent couplings in $\mathcal{V}$, if one can show that detailed balance is preserved under renormalization (which is the case in many nongravitational examples in condensed matter). If one is interested in getting close to general relativity with a small cosmological constant at long distances, detailed balance would clearly have to be broken, at least in the minimal theory. If that breaking happens only at the level of relevant deformations, the restrictive power of the detailed balance condition can still be useful for constraining the terms whose dimension equals that of the kinetic term.

Is it possible that the detailed balance condition could play a more physical role in our understanding of gravity? While this question remains open, one intriguing analogy seems worth pointing out: When gravity with anisotropic scaling satisfies the detailed balance condition, its path integral in imaginary time is formally analogous to the Onsager-Machlup theory of nonequilibrium thermodynamics [13, 14]. In this path-integral formulation of nonequilibrium systems, a collection of thermodynamic variables Φ^a is governed by the Onsager-Machlup action, given – up to surface terms – by

$$S = \frac{1}{2} \int dt \, d^D \mathbf{x} \left\{ \dot{\Phi}^a L_{ab} \dot{\Phi}^b + \frac{\delta W}{\delta \Phi^a} L^{ab} \frac{\delta W}{\delta \Phi^b} \right\}. \tag{1.6}$$

Here the Onsager kinetic coefficients L_{ab} represent a metric on the Φ^a space, $\frac{\delta W}{\delta \Phi^a}$ are interpreted as entropic forces, and the action W itself plays the role of entropy!

This analogy leads to a natural speculation, implicit in [1, 2], that the nature of gravity with anisotropic scaling is somehow entropic. It would be interesting to see whether this analogy can be turned into a coherent framework in which some of the intriguing recent ideas about the entropic origin of gravity [15] (also [16]) and cosmology [17] can be made more precise.

1.4 Causal dynamical triangulations and the spectral dimension of spacetime

In the study of quantum field theory, it is often useful to construct the system by a lattice regularization, and study the approach to the continuum limit using computer simulations.

In the context of quantum gravity, it is natural to define the lattice version by summing over random triangulations of spacetime. This approach works well in spacetimes of two Euclidean dimensions, where the system can be solved exactly in terms of matrix models. However, extending this success to higher dimensions has proven frustratingly difficult, with random triangulations typically yielding branched polymers or other phases with fractional numbers of macroscopic dimensions in the continuum limit.

In the past few years, a major breakthrough on this front has begun to emerge in the causal dynamical triangulations (CDT) approach to lattice gravity (see [18] for a review). In the CDT approach, the pathological continuum phases are avoided by changing the lattice rules slightly: The random triangulations that contribute to the partition sum are constrained to respect a preferred foliation of spacetime by fixed (imaginary) time slices. This seemingly innocuous change of the rules turns out to be relevant, in the technical RG sense: It leads to a different continuum limit, with much more attractive physical properties. The macroscopic dimension of spacetime in this continuum limit appears to be four, as is indicated by the measurement of the so-called "spectral dimension" d_s of spacetime [19] at the long distance limit in the lattice simulation (with $d_s = 4.02 \pm 0.1$ reported in [19]). This is a promising and exciting result, suggesting that perhaps for the first time, we might be close to sensible continuum results in lattice quantum gravity!

Clearly, the relevant change of the rules that makes all the difference in the lattice implementation, namely that spacetime is equipped with a preferred foliation structure, is very similar to the starting point of the analytic approach to quantum gravity with anisotropic scaling. It is natural to conjecture that *the CDT formulation of quantum gravity represents a lattice version of Lifshitz gravity with anisotropic scaling.*

The first nontrivial piece of evidence for this conjecture was presented in [3]. One of the surprises of [19] was not only that $d_s \approx 4$ at long distances, but also that at shorter distances, before the lattice artifacts kick in, d_s undergoes a smooth crossover to $d_s \approx 2$. How can the effective dimension of spacetime change continuously from four at long distances to two at short distances? An analytic explanation was offered in [3]: The spectral dimension is a precisely defined geometric quantity, and it can be calculated systematically in the continuum approach to quantum gravity with anisotropic scaling. In the mean-field approximation around the flat spacetime, the result is [3]

$$d_s = 1 + \frac{D}{z}. \tag{1.7}$$

Hence, if the gravity theory flows from a $z = 3$ UV fixed point to a $z = 1$ IR fixed point, the qualitative crossover of d_s observed in [19] is reproduced. The topological dimension of spacetime is always four, but the *spectral* dimension changes because of the anisotropic scaling at short distances.

This argument can be turned around, leading to a prediction: For example, in $2 + 1$ dimensions (not studied in [19]), the value of z required at the UV fixed point for power-counting renormalizability and UV completeness is $z = 2$, while the theory still flows to $z = 1$ in the IR. The Lifshitz gravity formula (1.7) then predicts that the CDT formulation of $2 + 1$ gravity should find a crossover from $d_s = 3$ at long distances to $d_s = 2$ at short distances. This prediction was beautifully confirmed in the CDT lattice approach in [20].
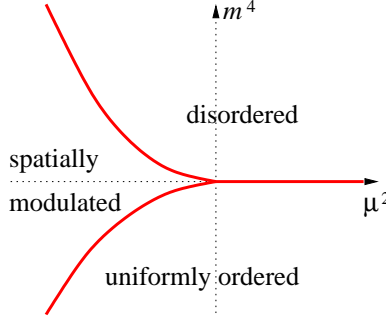
Figure 1. The mean-field phase diagram of the Lifshitz scalar theory (1.8).

1.5 Phases of gravity

Additional evidence for the conjecture relating CDT lattice gravity and the continuum gravity with anisotropic scaling comes from comparing the phase diagrams of the two approaches.

Recall first the phase structure of the Lifshitz scalar, first investigated in [21]. Including the relevant deformations, and a ϕ^4 self-interaction for stabilization, the theory is given by

$$S = \frac{1}{2} \int dt\, d^D \mathbf{x} \left\{ \dot{\phi}^2 - (\Delta\phi)^2 - \mu^2 \partial_i \phi \partial_i \phi - m^4 \phi^2 - \lambda \phi^4 \right\}. \tag{1.8}$$

With $\lambda > 0$, depending on the values of m^4 and μ^2, the theory can be in three phases. At positive μ^2, we obtain the standard disordered and and uniformly ordered phase, in which the vacuum expectation value of ϕ either vanishes or takes a constant nonzero value. At negative μ^2, a new, spatially modulated phase appears: In this phase, the vacuum condensate of ϕ is a periodic function along a spontaneously chosen spatial direction. The phase transition lines meet in the tricritical $z = 2$ point at $\mu = m = 0$. In the mean field approximation, the three phase transition lines joining at the tricritical point share a common tangent there [21]; this feature is erased by quantum corrections, and for tricritical Lifshitz points with multi-component order parameters.

In the CDT approach to quantum gravity in $3 + 1$ dimensions, three phases have also been observed, referred to as A, B and C [18]. One appears to give rise to a macroscopic de Sitter-like universe, while the other two attracted less attention at first, until recently [22]. The lines of phase transitions between these phases meet at a tricritical point, whose properties have not been explored in much detail on the lattice yet.

The phase diagram of gravity with anisotropic scaling [23] exhibits the same qualitative structure, with several phases organized around a multicritical point (see [23] for details). For simplicity, we illustrate this by considering the case of the projectable theory in $2 + 1$ dimensions, where the generic power-counting renormalizable potential is

$$\mathcal{V} = \alpha R^2 - \beta R + \gamma. \tag{1.9}$$

Up to a sign, the value of α can be absorbed into a rescaling of space versus time. The remaining sign determines whether we are in real or imaginary time. The terms with the β and γ couplings, which roughly play the role of the (inverse) Newton constant and the
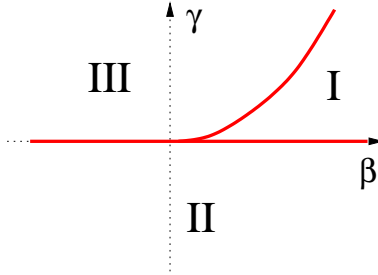
Figure 2. The mean-field phase diagram of gravity with anisotropic scaling in $2 + 1$ dimensions with the compact spatial slices $\Sigma = S^2$.

cosmological constant, represent relevant deformations. In the mean-field approximation, the phases are classified by assuming the FRW ansatz for the metric (with the spatial slices compact spatial slices $\Sigma = S^2$, as in CDT), and finding the vacuum solutions by solving the Friedmann equation. It turns out that – as in the Lifshitz scalar – there are three phases, which meet at the tritical $z = 2$ point with $\beta = \gamma = 0$. Amusingly, the phase transition lines also share a common tangent at the tricritical point, just as in the case of the Lifshitz scalar. We again expect that quantum corrections will modify this behavior, without changing the qualitative structure of the phase diagram.

The nature of the three phases can be analyzed in real or imaginary time. In real time, Phase I corresponds to a global de Sitter-like spacetime, Phase II describes a recollapsing cosmology with a big bang and a big crunch, while Phase III breaks time reversal spontaneously, with an expanding big-bang cosmology or a contracting cosmology with a big crunch. In imaginary time, Phase I yields a compact geometry on S^3 much like the shape found in [20], Phase II is a Euclidean bounce, and in Phase III, there are no solutions satisfying our maximally symmetric FRW ansatz.

This similarity between the phase diagram of quantum gravity with anisotropic scaling and the phase diagram found in the CDT approach represents further evidence [23] for our conjecture that these two approaches to quantum gravity are intimately related. Another universal lesson emerging from our analysis of the phase structure of quantum gravity with anisotropic scaling in [23] is that spatially modulated phases of gravity should be possible.

2 General Covariance in Gravity with Anisotropic Scaling

In order to eliminate the extra scalar polarization of the graviton, gravity with anisotropic scaling which enjoys an extended gauge invariance was proposed in [4]. The gauge symmetry in question is an extension of the foliation-preserving diffeomorphisms by an Abelian gauge symmetry, and can be interpreted as a nonrelativistic form of general covariance. The number of independent symmetries per spacetime point is the same as in general relativity, with the Abelian symmetry playing the role of linearized spacetime-dependent time reparametrizations. This extended symmetry preserves the preferred spacetime foliation and the privileged role of time, but eliminates the scalar polarization of the graviton.

2.1 Fields and symmetries

We start with the minimal projectable theory reviewed in Part 1. It was noticed in [2] that at $\lambda = 1$, this theory exhibits in the linearized approximation around flat spacetime an enhanced symmetry, which acts only on the shift vector,

$$\delta N_i = \partial_i \alpha, \tag{2.1}$$

with $\alpha(\mathbf{x})$ a time-independent local symmetry generator. Promoting this symmetry to a spacetime-dependent gauge symmetry of the full nonlinear theory will lead to our desired nonrelativistic general covariance.

Extending (2.1) to a gauge symmetry requires new fields beyond the minimal gravity multiplet g_{ij}, N_i and $N(t)$. Already at the linearized level, we need to introduce a new field A which transforms under $\alpha(\mathbf{x}, t)$ as the time component of an Abelian gauge field. In the interacting theory, this transformation rule becomes

$$\delta A = \dot{\alpha} - N^i \partial_i \alpha. \tag{2.2}$$

The new field A, and the new gauge symmetry α, have an elegant and geometric interpretation [4] in the context of a nonrelativistic $1/c$ expansion of relativistic gravity: A is simply the subleading term in the $1/c$ expansion of the relativistic lapse function, and α is the subleading, linearized part of spacetime-dependent time reparametrizations.

Unfortunately, in dimensions greater than $D = 2$, this is not the whole story. When $D > 2$, the linearized symmetry (2.1) does not extend to a symmetry of the interacting, nonlinear theory, and therefore cannot be straightforwardly gauged. In order to fix this obstruction, a new field ν was introduced in [4]. This "Newton prepotential" transforms as a Goldstone field,

$$\delta \nu = \alpha. \tag{2.3}$$

The introduction of the Newton prepotential allows (2.1) to be extended to a symmetry of the nonlinear theory, which can then be gauged by the standard coupling to the gauge field A. Unlike the rest of the gravity multiplet, the Newton prepotential does not appear to have a natural geometric interpretation in terms of the $1/c$ expansion in the metric formulation of relativistic gravity.

2.2 The Lagrangian and Hamiltonian formulations

The systematic construction of an action invariant under the extended gauge symmetries leads to the following minimal theory [4],

$$S = \frac{2}{\kappa^2} \int dt \, d^D \mathbf{x} \, \sqrt{g} \left\{ N \left[K_{ij} K^{ij} - K^2 - \mathcal{V} + \nu \, \Theta^{ij} \left(2 K_{ij} + \nabla_i \nabla_j \nu \right) \right] - A \left(R - 2\Omega \right) \right\}. \tag{2.4}$$

Here Θ^{ij} a short-hand for $\Theta^{ij} = R^{ij} - \frac{1}{2} g^{ij} R + \Omega g^{ij}$, and Ω is a new relevant coupling constant, of the same dimension as the cosmological constant Λ. It controls the scalar curvature of the spatial slices in the preferred foliation $\mathcal{F}$ of spacetime, and it makes sense to refer to Ω as the "second cosmological constant." The form of the potential $\mathcal{V}$ is again unconstrained by the symmetries, just as in the minimal theory.

The theory can also be rewritten in the Hamiltonian formalism [4], which offers a more systematic way for studying the gauge symmetry structure and counting the number of propagating degrees of freedom without having to resort to sometimes unreliable linearizations around a chosen background. The Hamiltonian constraint algebra exhibits an intriguing mixture of first- and second-class constraints, and confirms that the theory propagates only the tensor graviton polarizations. The scalar graviton mode is seen as a gauge artifact of nonrelativistic general covariance.

2.3 Comparing to general relativity in the infrared

Since the spectrum of propagating gravitons – and gravitational waves – in the long-distance limit of our gravity with nonrelativistic general covariance matches that of general relativity, it is natural to extend this comparison to the long-distance limits of the full nonlinear theories.

Some first steps in this direction were made in [4]. First, a simple conceptual argument implies that the Schwarzschild spacetime is an exact solution of the infrared limit of our theory. This bodes well for the standard tests, since it suggests that in the infrared regime, the β and γ parameters of the PPN formalism take their relativistic value, equal to one.

The equation of motion associated with the variation of A constrains the spatial scalar curvature to be constant, $R = 2\Omega$. At first, it might seem that this equation might be difficult to reconcile with the existence of interesting cosmological solutions. However, this issue can be avoided in several different ways, and interesting cosmological solutions can be found [4]. In fact, the theory has a phenomenologically attractive feature: it seems to prefer cosmologies whose preferred spatial slices are flat.

Perhaps the biggest challenge for this program is to explain why the infrared limit should exhibit Lorentz invariance, to the high level of accuracy required by observations. While the theory may naturally flow to $z = 1$ at long distances, different species of low-energy probes may experience distinct effective limiting speeds of propagation, not equal to the speed of light. Setting all these speeds equal to c would represents a rather unpleasant amount of fine tuning. While this problem remains unsolved in the theory with nonrelativistic general covariance as well, it is intriguing that – unlike in the minimal theory – global Lorentz symmetries of the flat spacetime can be embedded into the extended gauge symmetry of our generally covariant theory [4].

3 Conclusions

If one's agenda is to construct a theory with anisotropic scaling which resembles general relativity in the infrared, the generally covariant model of [4] appears to be a step in the right direction, since its extended gauge symmetry eliminates the scalar graviton from the theory, leaving only the physical tensor polarizations. The resulting infared limit has the Schwarszchild geometry as an exact solution, suggesting that the theory is likely compatible with the standard solar-system tests.

The price paid is the introduction of the rather mysterious Newton prepotential ν in [4]. This field does not appear to have a clear geometric interpretation in the $1/c$ expansion of the standard metric formulation of relativistic gravity. Moreover, the introduction of ν leads

to a new proliferation of gauge invariant terms that can appear in the action, both for pure gravity and in its coupling to matter [24, 25]. Clearly, a better understanding of the role of the Newton prepotential is desirable before one can seriously discuss detailed phenomenological constraints on models of gravity with nonrelativistic general covariance.

Acknowledgments

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